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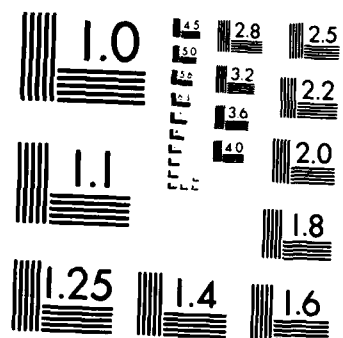
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# AFOSR-TR- 86 - 0897

PROGRESS REPORT NO. 2

GRANT NO: AFOSR 84-0240

PERIOD: 16 JULY 1984 TO 16 DECEMBER 1985

PRINCIPAL INVESTIGATOR: Stephen L. Campbell

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## PROJECT DESCRIPTION

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFOSR)  
MEMORANDUM FOR THE DIRECTOR

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Chief, Technical Information Division

Implicit systems of differential equations of the form  $F(t, y, y') = 0$  naturally arise in many circuit and control problems, economic models, and the solution of partial differential equations by the method of lines. Implicit systems are also called singular, differential-algebraic, semi-state, constrained, and descriptor. The theory is well understood, and numerical codes exist, for index zero, index one, and linear constant coefficient problems. Higher index systems occur in circuit and control problems. The numerical and analytic behavior of such higher index systems is not well understood and is incomplete. It has recently been shown that traditional methods, such as backward differences, need not work on higher index systems. Good characterizations of the solution manifold are often difficult to obtain. This research project is to study the numerical and analytic solution of higher index implicit differential equations. Applications will be made to circuit theory, control theory, and the analysis of numerically ill-conditioned index one systems.

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## PUBLICATIONS

During the report period the following papers were written as part of this research effort;

- P1. S. L. Campbell, *Rank deficient least squares and the numerical solution of linear singular implicit systems of differential equations*, in Linear Algebra and Its Role in Systems Theory, AMS Contemporary Mathematics Series, Vol 47(1984), 51-64. (Invited paper)
- P2. S. L. Campbell, *Index two linear time varying singular systems of differential equations II*, Circuits, Systems, and Signal Processing (to appear).
- P3. S. L. Campbell, *Nonlinear time-varying generalized state-space systems: an overview*, Proc. 23 IEEE Conference on Decision and Control, (1984), 268-276 (Invited paper)
- P4. S. L. Campbell, *Consistent initial conditions for linear time varying singular systems*, Proc. 1985 Mathematical Theory of Networks and Systems, Stockholm, 1985 (to appear) (Invited paper)
- P5. K. D. Clark, *Difference methods for the numerical solution of time-varying singular systems of differential equations* SIAM J. Algebraic & Discrete Methods (to appear)
- P6. S. L. Campbell, *A general form for solvable linear time varying singular systems of differential equations*, (preprint) 1985
- P7. K. D. Clark, *The numerical solution of some higher index time varying semi-state systems*, (preprint) 1985.

Additional references of the principal investigator [Ci] and others [Ri] appear at the end of this report.

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## PROGRESS TO DATE

If a system of differential equations is written as

$$u' = f(u, v, t) \quad (1a)$$

$$0 = g(u, v, t) \quad (1b)$$

then the system (1) is called index one if  $\frac{\partial g}{\partial v}$  is nonsingular. In this case the algebraic constraint (1b) may be solved for  $v$ , to give  $v = \phi(u, t)$  and (1a) becomes an explicit equation  $u' = f(u, \phi(u, t), t)$ . However, in some control applications [C1], [C7], [R1], [R5] the matrix  $\frac{\partial g}{\partial v}$  is always singular. These problems are called higher index problems. Definitions of index for the more general system

$$F(x, x', t) = 0 \quad (2)$$

appear in [C5]. Traditionally (1) has been numerically solved using backward difference formulas (BDF) [R2], [R3]. Recently, it has been shown that BDF's need not converge both for nonlinear [C5] and linear time varying [R3]

$$A(t)x'(t) + B(t)x(t) = f(t) \quad (3)$$

higher index systems. Prior to the work of the principal investigator, no general method for the numerical solution of (3) existed.

An approach to numerically solve (3) was proposed in [C4]. However, numerous questions on the practical implementation and theoretical implications of this method remained unresolved. Some of these problems have now been resolved.

The key idea in [C4] was as follows. Suppose that (3) is solvable on an interval  $I$ . That is, for every sufficiently smooth input  $f$  there is at least one solution  $x$  and the solutions for a given  $f$ , are uniquely determined by their value at any time  $t \in I$ . From [C2] it is known that the solution can depend not only on  $A, B, f$  but also their derivatives. Assume that  $A, B, f$  are sufficiently smooth on  $I$  and at time  $\hat{t} \in I$ , let  $\{A_i\}, \{B_i\}, \{x_i\}, \{f_i\}$  be the Taylor coefficients of  $A, B, x$ , and  $f$  respectively. These functions do not have to be infinitely differentiable, just sufficiently differentiable. Then for any  $j > 0$ , any  $\hat{t} \in I$ , equation (3) implies that the coefficients satisfy

$$\begin{bmatrix} A_0 & . & . & . & . \\ A_1 + B_0 & 2A_0 & . & . & . \\ A_2 + B_1 & 2A_1 + B_0 & . & . & . \\ . & . & . & . & . \\ A_{j-1} + B_{j-2} & 2A_{j-2} + B_{j-3} & . & . & jA_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_j \end{bmatrix} = - \begin{bmatrix} B_0 \\ B_1 \\ . \\ . \\ B_{j-1} \end{bmatrix} x_0 + \begin{bmatrix} f_0 \\ f_1 \\ . \\ . \\ f_{j-1} \end{bmatrix} \quad (4)$$

or more compactly

$$A_j x_j = f_j. \quad (5)$$

In [C4] it is shown that for all known classes of solvable systems that if  $j$  is appropriately chosen, then (5) uniquely determines  $x_1$  even though  $x_j$  is not unique. Since  $x_1 = x'(\hat{t})$  it is now possible to define a method to integrate (3).

This general approach was refined and improved by the principal investigator in [P1]. This paper made several contributions. First, it examined the numerical considerations in solving (5) and showed how to do so in a numerically reliable manner. Secondly it greatly reduced the computational effort of the method proposed in [C4] by showing how (5) could be used to define an explicit differential equation  $x' = R(t)x + r(t)$  at each time step. This made possible the definition of both Adams and Runge-Kutta methods. Due to the smaller sized coefficient matrices involved, these methods were more efficient than the higher order Taylor methods of [C4].

One previous difficulty with many of the approaches for solving (3), including the BDF, has been the lack of general methods for finding the manifold of consistent initial conditions. In [P4] the principal investigator showed that for many classes of solvable systems, that the equations (5) could be used to develop an algorithm for calculating the consistent initial conditions. This is the first general method for doing so based strictly on the coefficients of  $A$ ,  $B$ ,  $f$  and not requiring a series of time-varying coordinate changes. While the results reported so far made substantial progress on the analysis of the linear time varying system (3), several major difficulties remained. Among these were the lack of either any explicit criteria for solvability or good descriptions of how to approximate one linear system by another. Substantial progress has been made in this direction within the last few months in [P6].

Prior characterizations of solvability, often amounted to being able to solve the problem. In [P6], the principal investigator gave the first explicit, verifiable, characterization of solvability in the cases when  $A, B, f$  are either real analytic or infinitely differentiable. Necessary conditions are given if



$A, B, f$  are  $2n$ -times differentiable. These characterizations are stated directly in terms of pointwise rank conditions involving the arrays in (4). Thus they may be verified without performing any coordinate changes.

One consequence of [P6] is a proof that the numerical method of [P1] works for all smooth linear time varying implicit solvable systems (3).

Another consequence of [P6] is a characterization of when the solution manifolds of two linear time varying singular systems are close. This is the first general approximation result to appear in the linear time varying case.

The results to date, especially [P1], [P3], [P6] have finally put the numerical and analytic solution of (3) on a firm footing. Techniques, for the first time, are now available to study the approximation and simplification of general linear time varying implicit systems.

Petzold and Gear [R3], [R4] have shown that most higher index systems (3) can be changed, at least locally, to an index two system by the use of linear time varying coordinate changes. Thus the index two case is of considerable interest. In [C2] the principal investigator had given both sufficient conditions under which a linear time varying singular system was solvable and a characterization of the solutions. In [P2] the principal investigator weakened the assumptions of [C2] and obtained a similar result. A series of examples in [P2] showed that additional weakening is probably not possible. These examples also shed light on the behavior of systems which are index two but have higher index in a different coordinate system. These examples will prove useful in testing numerical algorithms.

There are two natural types of coordinate changes to make when studying (3); letting  $x = Q(t)y$  and multiplication of (3) on the left by  $P(t)$ . That neither of

these effect the analysis of [P1], [P4], [P6], [C4] is one reason that approach is so useful.

It is known that the question of whether BDF's converge is unaltered by time varying P and constant Q but can be altered by time varying Q. Clark has defined a modified backwards difference formula (MBDF) in [P5]. This method has several interesting properties. For linear time invariant systems and those in the standard canonical form [C6] it is the same as the BDF. However, the convergence properties of the MBDF are unchanged by constant P and time varying Q, but can be altered by time varying P. Thus the MBDF methods provide a nice symmetry to the theory. The methods of [P1], [P4], [C4] are unaffected by time varying P and Q. Additional results on when the BDF and MBDF methods work is given in [P7]. This unifies and extends some of the work of [R1].

The current interest in implicit systems within the mathematical and engineering communities is illustrated by the recent special sessions held on this topic. Several of these are described in the travel section. In addition, the paper [P2] will appear in a special issue of Circuits Systems & Signal Processing on semistate equations.

#### CURRENT RESEARCH

The application of the reported research to linear time varying control and circuit problems is under investigation. Preliminary results indicate that the special structure of many control problems can be exploited to greatly speed up the numerical methods of [P1]. The use of the approximation results from [P6] to simplify specific implicit control problems is also under investigation. The extension of these results to nonlinear problems is being examined.

## TRAVEL

Travel funds from this grant were used to enable the principal investigator to attend the Society for Industrial and Applied Mathematics (SIAM) 1984 National Summer Meeting in Seattle, Washington, July 16-20. A paper concerning this research was presented. The meeting provided an opportunity to discuss the proposed research with experts from numerical analysis, control theory, and differential equations.

July 30 - August 3, 1984; the principal investigator attended and gave an invited paper [P1] at the American Mathematical Society (AMS) - SIAM Research Conference on "Linear Algebra and Its Role in Systems Theory". Researchers from Engineering and Mathematics with an interest in control theory, systems theory, linear algebra, and numerical linear algebra were present. Recent results by the principal investigator on the linear algebra problems that arise in the numerical solution of the implicit systems (3) were presented.

December 12-14, 1984; the principal investigator attended and gave an invited paper [P3] at the 23<sup>rd</sup> IEEE Conference on Decision and Control as part of a special session on generalized state space systems.

June 9-15; the principal investigator attended and gave an invited talk [P4] at the 1985 Mathematical Theory of Networks and Systems Conference in Stockholm, Sweden.

June 18-23, 1985; the principal investigator visited the University of Bologna, Italy. He gave a seminar on recent results on singular systems and had mathematical discussions with Professor Angelo Favini about operator theoretic questions involving implicit systems of differential equations. While considering these questions, the principal investigator was able to prove, by operator theoretic means, one of the key lemmas in [P6].

June 24 - 26; the principal investigator attended the 1985 SIAM National meeting in Pittsburgh, PA. He gave an invited talk in the special session on differential-algebraic equations.

#### OTHER PROJECT PERSONNEL

##### Research Assistant:

The Research Assistant; Mr. Kenneth Clark, is working on a Ph.D. in Applied mathematics under the direction of the principal investigator. Mr. Clark has recently developed some nice results on the relationship between the convergence of backward difference formulas and the types of transformations needed to put an implicit system into a canonical form. These results are reported in [P5], [P7]. He is expected to graduate in June, 1986. An abstract of his thesis follows.

#### Abstract of Research Assistant Ph.D. Thesis

TITLE: Difference Methods for the Numerical Solution of Nonautonomous Singular Linear Systems.

ABSTRACT. Several aspects of the numerical and analytical treatment of singular linear systems

$$A(t)x' + B(t)x = f(t)$$

are considered. Here  $A(t)$  is assumed to be identically singular, and the differential equation is considered on the closed interval  $[0, T]$ . First, a class of modified difference schemes is derived for the numerical solution of the system. These methods are analyzed on several classes of problems which frequently arise, and in particular their convergence and stability is proved for the classes considered. The classes we examine include those transformable to

constant coefficients, implicitly defined index one systems, and index two systems in semi-explicit form. We show that all of the classes investigated in this paper can be treated in a more general framework involving a special case of the standard canonical form for time varying singular linear systems, and that the structure of the form is the key element in the analysis.

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- C1. S. L. Campbell, *Singular Systems of Differential Equations*, Pitman, 1980.
- C2. S. L. Campbell, *Index two linear time-varying singular systems of differential equations*, SIAM J. Algebraic and Discrete Methods, 4(1983), 237-243.
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- R1. K. Brennan, *Stability and convergence of difference approximations for higher index differential algebraic systems with applications in trajectory control*, Ph.D. Thesis, Univ. of Calif., Los Angeles 1983.
- R2. C. W. Gear, *Simultaneous numerical solution of differential-algebraic equations*, IEEE Trans. Circuit Theory, CT-18, (1971), 89-95.
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